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Composition series and colour symmetry point groups

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Abstract. The idea of composition series that exist among the 32 crystallographic point groups is explored in rederiving the 106 colour symmetry point groups (58 magnetic, 18 polychromatic, 30 multicolour groups) which are associated with the distinct irreducible representations of the crystallographic point groups employing the concept of colour generators. The colour symmetry group associated with an irreducible representation of G_i is generated either from the maximal normal subgroup G_{i+1} or from an appropriate colour symmetry group associated with G_{i+1}. Simultaneous generation of various colour symmetry point groups that may exist for all the point groups contained in a chosen composition series is a feature of this present method. The elegance of this method and its physical significance are also briefly discussed.

1. Introduction

In contemporary group theoretical research, we note that various aspects of the work are from time to time, recapitulated and systematised. For instance, the concept of composition series which gained fundamental importance in physics has been explored recently in a variety of ways by several investigators. A peep into the past is not out of place: for instance in constructing the irreducible representations (IRS) of space groups using the allowable irreducible representations (AIRS) of the little groups in conjunction with the solvability property (Raghavacharyulu 1958, 1961, Bradley and Cracknell 1972), in deriving the IRS of the crystallographic point groups with the help of the generating relations (Ramachandra Rao 1973), in suggesting and describing an alternative method of studying the magnetic properties of crystals and enumerating the number of independent constants required to describe a chosen magnetic property with the help of the IRS of the factor groups contained in a composition series (Krishnamurty *et al* 1977) etc.

Regarding the construction of colour symmetry point groups, the concept of anti-symmetry introduced by Shubnikov (1951) has been used to obtain the 58 magnetic (double colour) point groups in a variety of ways (Tavger and Zaitsev 1956, Hamermesh 1962, Bhagavantam and Pantulu 1964, Tinkham 1964, Krishnamurty and Gopala Krishna Murty 1969). Quite a large number of papers reporting studies on the various generalisations of the concept of antisymmetry have appeared during the past two decades. Belov and Tarhova's (1956) work on polychromatic symmetry, Indenbom *et al*'s (1960) work on the construction of 18 polychromatic point groups, Niggli and Wondratschek (1960) and Wondratschek and Niggli's (1961) work on the crypto-symmetry are some of the important publications in this area.

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Zamorzaev's innovative work (1967) on the quasisymmetry (P-symmetry) groups enveloped all the important earlier phenomenon of antisymmetry, coloursymmetry and cryptosymmetry into its fold. The 58 magnetic groups, the 18 polychromatic groups were shown, with the help of the fundamental quasisymmetry theorem, to be full P-symmetry minor groups with appropriate crystallographic point groups as generators and suitable permutation (cyclic) groups of orders 2, 3, 4 or 6 as the group of indices P.

Krishnamurty *et al* (1978) established a general method of obtaining quasisymmetry (P-symmetry) groups as semi-direct products. These authors also proposed a new method of associating the obtained minor quasisymmetry groups with the IRs of the generator groups using the ideas of little groups and their AIRs which is different from that of the earlier investigators (Niggli and Wondratschek 1960). In a recent paper, the author (Rama Mohana Rao 1982) has constructed 30 minor quasisymmetry groups as semi-direct products with crystallographic point groups as generators and associated them with the 23 two-dimensional (2D) and 7 distinct three-dimensional (3D) IRs of the generator groups. These minor quasisymmetry groups can be viewed as the multicolour groups when the basic permutations $p_i \in P$ are identified with suitable colours R_i, $i = 2$, 3, 4 or 6.

The aim of the present paper is to utilise the idea of composition series that exist among the 32 crystallographic point groups (Lomont 1959) to bring a host of the earlier results on to a common footing in an elegant manner invoking the concept of colour generators. Whereas the simultaneous generation of the various colour symmetry point groups that may exist for all the point groups contained in a considered composition series is the achievement of this work, obtaining the colour symmetry point group associated with the IR of G_i generated either from the maximal normal subgroup G_{i+1} or from an appropriate colour symmetry group associated with G_{i+1} is a new feature of this present method.

In § 2, the 37 composition series needed to enumerate the 106 colour symmetry point groups are constructed and are tabulated. In § 3 the method employed to obtain the colour symmetry point groups from a considered composition series is discussed. This is illustrated with the help of one or two series and the results tabulated in respect of each of the 37 series, in the standard notation established earlier in respect of the magnetic, polychromatic and multicolour point groups respectively by Shubnikov and Belov (1964), Indenbom *et al* (1960), Rama Mohana Rao (1982). The underlying physical significance of this method and a brief discussion of the significant features of this work *vis-a-vis* the results obtained are provided in §§ 4 and 5 respectively.

2. Formulation of composition series

It will be helpful here to recall the concept of the composition series before the 32 crystallographic point groups are expressed in terms of the series needed for our purpose.

Let $G = G_0$ be a finite group of known order, G_1 be a maximal normal subgroup of G_0, G_2 to that of G_1 and in general G_{i+1} to G_i. Since $G = G_0$ is of finite order, this process ultimately terminates with the group consisting of the identity element alone i.e. $G_0 \supset G_1 \supset G_2 \supset \ldots \supset G_i \supset G_{i+1} \supset \ldots \supset G_s = E$. Such a series is called a composition series of G_0.

Because a maximal normal subgroup of a group is not unique, we obtain several such composition series for a group G_0. The factor groups G_k/G_{k+1} are not unique (but for isomorphism).

In what follows, the 37 distinct composition series just sufficient for the purpose of obtaining the 106 colour symmetry point groups are constructed and are tabulated in table 1. The nomenclature adopted for the point groups in the considered composition series is that of the Hermann-Maüguin (International) notation.

Table 1. Crystallographic point groups in terms of the chosen composition series.

1	$m3m \supset 432 \supset 23 \supset 222 \supset 2 \supset 1$
2	$m3m \supset \bar{4}3m \supset 23 \supset 222 \supset 2 \supset 1$
3	$m3m \supset m3 \supset 23 \supset 222 \supset 2 \supset 1$
4	$m3m \supset m3 \supset mmm \supset mm2 \supset m \supset 1$
5	$m3m \supset m3 \supset mmm \supset 222 \supset 2 \supset 1$
6	$m3m \supset m3 \supset mmm \supset 2/m \supset \bar{1} \supset 1$
7	$4/mmm \supset 422 \supset 222 \supset 2 \supset 1$
8	$4/mmm \supset 422 \supset 4 \supset 2 \supset 1$
9	$4/mmm \supset \bar{4}2m \supset \bar{4} \supset 2 \supset 1$
10	$4/mmm \supset \bar{4}2m \supset 222 \supset 2 \supset 1$
11	$4/mmm \supset \bar{4}2m \supset mm2 \supset 2 \supset 1$
12	$4/mmm \supset 4mm \supset mm2 \supset m \supset 1$
13	$4/mmm \supset 4mm \supset 4 \supset 2 \supset 1$
14	$4/mmm \supset 4/m \supset 2/m \supset m \supset 1$
15	$4/mmm \supset 4/m \supset 2/m \supset \bar{1} \supset 1$
16	$4/mmm \supset 4/m \supset 4 \supset 2 \supset 1$
17	$4/mmm \supset 4/m \supset \bar{4} \supset 2 \supset 1$
18	$4/mmm \supset mmm \supset 2/m \supset 2 \supset 1$
19	$4/mmm \supset mmm \supset 2/m \supset m \supset 1$
20	$6/mmm \supset 622 \supset 6 \supset 3 \supset 1$
21	$6/mmm \supset 622 \supset 6 \supset 2 \supset 1$
22	$6/mmm \supset \bar{6}2m \supset 32 \supset 3 \supset 1$
23	$6/mmm \supset 6mm \supset 6 \supset 3 \supset 1$
24	$6/mmm \supset 6/m \supset \bar{6} \supset 3 \supset 1$
25	$6/mmm \supset 6/m \supset \bar{6} \supset m \supset 1$
26	$6/mmm \supset 6/m \supset 6 \supset 2 \supset 1$
27	$6/mmm \supset 6/m \supset \bar{3} \supset \bar{1} \supset 1$
28	$6/mmm \supset 6/m \supset 2/m \supset 2 \supset 1$
29	$6/mmm \supset \bar{3}m \supset 32 \supset 3 \supset 1$
30	$6/mmm \supset 622 \supset 32 \supset 3 \supset 1$
31	$6/mmm \supset 6mm \supset 3m \supset 3 \supset 1$
32	$6/mmm \supset \bar{6}2m \supset 3m \supset 3 \supset 1$
33	$6/mmm \supset \bar{6}2m \supset \bar{6} \supset 3 \supset 1$
34	$6/mmm \supset 6/m \supset \bar{3} \supset 3 \supset 1$
35	$6/mmm \supset \bar{3}m \supset 3m \supset 3 \supset 1$
36	$6/mmm \supset \bar{3}m \supset \bar{3} \supset 3 \supset 1$
37	$6/mmm \supset \bar{3}m \supset \bar{3} \supset \bar{1} \supset 1$

3. The method

The elegant method employed in the present paper for the construction of all the colour symmetry point groups is illustrated for each of the categories namely the magnetic (double colour), polychromatic and multicolour point groups with the help of one or two composition series.

Case (a): Magnetic (double colour) point groups

The magnetic point groups G_i' associated with the point groups G_i are the type III Shubnikov point groups M (Bradley and Cracknell 1972) and they are derived in this section from the maximal normal subgroup G_{i+1}, of index 2, of the group G_i. Thus G_i' is induced from G_{i+1} by considering the semi-direct or quasi semi-direct product (as the case may be) of G_{i+1} with the colour group E, R_2g. g is the generator that generates G_i from G_{i+1} and R_2 is the suitable colour changing operation (or a permutation on two symbols) associated with the generator g to form the appropriate colour generator R_2g.

However, it may be noted that, if the maximal normal subgroup G_{i+1} is of index 3, then no magnetic group is obtained for G_i from G_{i+1}. This is because the generator g in this case is of order 3 and does not commute with all the elements of G_{i+1}, i.e. $gx \neq xg$ but is equal to xg^2, where x is any element of order 2 of G_i other than g—$x \in (G_i \backslash g)$. If the generating element is the same in two different composition series for the considered G_i, then no new magnetic group G_i' is obtained for G_i in the later series. The remaining magnetic point groups for G_i, if any, can be obtained by considering other composition series involving that group with the other maximal normal subgroups $\tilde{G}_{i+1}$ isomorphic with G_{i+1}.

As an example consider the series (1)

$$m3m \supset 432 \supset 23 \supset 222 \supset 2 \supset 1. \tag{i}$$

The point group 2 is generated from 1 by C_{2z}. Hence the appropriate colour generator is R_2C_{2z}, where R_2 is the colour changing operation with $R_2^2 = E$ and in the language of indices (Zamorzaev 1967) is given by (12). The colour group generated by R_2C_{2z} is E, R_2C_{2z} : E, C_{2z} (12) and the magnetic variant associated with 2 is obtained by taking the semi-direct product of 1 with (E, R_2C_{2z}). We call the resultant group 2′. Similarly the colour generator from 2 to 222 can be taken as R_2C_{2x} and a colour group generated by this is E, R_2C_{2x}. The semi-direct product of 2 with E, R_2C_{2x} gives the magnetic variant of 222: E, C_{2z}, R_2C_{2x}, R_2C_{2y} which is denoted as 2′2′2. This procedure when extended to the group 23 does not give rise to any magnetic variant since C_3 is the generating element and it does not commute with other point group operations of order 2 of 222 as explained earlier. However, the magnetic point group associated with 432 can be obtained as usual by considering the semi-direct product of 23 with the colour group generated by R_2C_2 as before—thus giving rise to 4′32′. In a similar way the magnetic variant of $m3m$ can be evaluated from 432 by realising that i is the generating element of $m3m$ from 432 and E, R_2i is the colour group generated by this generator. We denote this variant by $m'3m'$. Thus from the first composition series, the magnetic point groups 2′, 2′2′2, 4′32′ and $m'3m'$ are obtained and they are given in column 1 of table 2 before that series.

Case (b): The polychromatic point groups

The alternative method of generating the 18 polychromatic point groups discussed in this section with the help of the appropriate colour generators and with the suitably chosen composition series differs from that of the earlier investigators (Indenbom *et al* 1960, Krishnamurty and Appalanarasimham 1972). This is done by adopting two procedures: one for those groups whose colour value is 3 (with R_3 as the colour changing operation $\ni R_3^3 = E$) and the other for those with colour value 4 or 6 (using R_4 and R_6).

For the polychromatic groups of the first category with colour value 3, the maximal normal subgroup G_{i+1} of G_i can always be chosen in such a way that the index of G_{i+1} to G_i is equal to 3 and in all the cases, the polychromatic point groups are obtained in much the same way as a magnetic point group is obtained by identifying the appropriate generator (of order 3) and forming the semi-direct product of G_{i+1} with the colour group generated by the colour generator. As an example consider the same series (i). We know that 23 contains a pair of 1D complex IRs and the factor group $23/222$ (in this series) $\simeq 3$ with the symmetry element C_{31}^+ of order 3 as the generator from 222 to 23. The colour group generated by the appropriate colour generator $R_3C_{31}^+ : C_{31}^+$ (123) is $E, R_3C_{31}^+, R_3^2C_{31}^- : E, C_{31}^+$ (123), C_{31}^- (132). Now the polychromatic point group associated with 23 can be obtained by taking the semi-direct product of 222 with the above colour group. Denote the resultant colour symmetry group $G_i^{(3)}$ as $3^{(3)}/2$.

When a polychromatic point group whose colour value is either 4 or 6 is to be generated, the normal subgroup G_{i+1} of G_i must be chosen to be of index 4 or 6 respectively. But in the case of any crystallographic point group G_i, it is not possible to find a maximal normal subgroup G_{i+1} of index 4 or 6 in a composition series. In such a difficulty the polychromatic point group $G_i^{(p)}$, $p = 4$ or 6, is generated, in a novel way, here from another appropriate polychromatic group $G_{i+1}^{(p')}$, $p' = 2$ or 3, associated with the maximal normal subgroup G_{i+1}, instead of from the point group G_{i+1} as was done earlier.

To illustrate this method, consider the series

$$4/mmm \supset 422 \supset 4 \supset 2 \supset 1$$

and

$$6/mmm \supset 622 \supset 6 \supset 3 \supset 1.$$

Here the polychromatic point group of colour value 4 associated with 4 is generated not directly from the normal subgroup of index 4 namely from the point group 1 as is done by the earlier investigators (Indenbom *et al* 1960, Krishnamurty and Appalanarasimham 1972), but is obtained from the double colour (magnetic) point group $G_{i+1}^{(2)} = 2'$ associated with 2. This is done by identifying $2'$ as† $\tilde{2}' = E, R_4^2C_{2z} : E, C_{2z}$ (13) (24) and realising the colour group generated by the colour generator $R_4C_{4z}^+ : C_{4z}^+$ (1234) as the group by modulus‡ $C_4^{(4)}$ (mod C_2): $E, \tilde{C}_{4z}^+$ (1234) and taking the quasi semi-direct product (Shubnikov and Koptsik 1974) of $\tilde{2}'$ with $C_4^{(4)}$ (mod C_2) and combining the resultant elements.

Similarly the polychromatic group $6^{(6)}$ associated with 6 can be obtained from the later series by considering the semi-direct product of the group $\tilde{3}^{(3)}$: $E, R_6^2C_3^+, R_6^4C_3^-$ where R_6 stands for (123456) and R_6^2 for (135) (246), with the double colour group† $\tilde{2}' = E, R_6^3C_2 : E, C_2$ (14) (25) (36) generated by the colour generator $R_6^3C_2 \simeq R_2C_2$. The results obtained are provided in table 2.

† $\tilde{2}'$ can be viewed as a double colour group isomorphic with the magnetic point group $2' = E, R_2C_{2z} : E, C_{2z}$ (12) generated with the point group 2 and is expressed as E, C_{2z} (13) (24) or E, C_{2z} (14) (25) (36) in some non-standard setting of the point group 2.

‡ E, C_{4z}^+ (1234) is the colour symmetry group $C_4^{(4)}$ (mod C_2) associated with the elements E, C_{4z}^+ (which occur as the coset reprentatives in the decomposition of C_4/C_2) obtained in expressing the point group C_4 as the group by modulus: $C_4 = C_2 \circ C_4$ (mod C_2).

Table 2. Colour symmetry point groups associated with different point groups in the considered composition series.

Composition series number	(1)	(2)	(3)	(4)
1	$2'$	$3^{(3)}/2$	O''	T'''
	$2'2'2$			O'''
	$4'32$			
	$m'3m'$			
2	$\bar{4}'3m'$		T_d''	T_d'''
	$m'3m$			
3	$m'3$			${}^1O_h'''$
	$m3m'$			${}^2O_h'''$
4	m'			
	$m'm2'$			
	mmm'			
5	$m'm'm'$	$\bar{6}^{(3)}/2$	${}^1O_h''$	${}^1T_h'''$
		$\bar{6}^{(6)}/2$	${}^2O_h''$	${}^2T_h'''$
6	$\bar{1}'$			
	$2'/m'$			
	$m'm'm$			
7	$4'22'$			
	$4/m'm'm'$			
8	$4'$ (*)	$4^{(4)}$ (*)	D_4''	
	$42'2'$			
9	$\bar{4}'$ (*)	$\bar{4}^{(4)}$ (*)	D_{2d}''	
	$\bar{4}2'm'$			
	$4'/m'm'm$			
10	$\bar{4}'2m'$			
11	$m'm'2$			
	$\bar{4}'m2'$			
12	$4'mm'$			
	$4/m'mm$			
13	$4m'm'$		C_{4v}''	
14	$4'/m$ (*)	$4^{(4)}/m$ (*)		
15		$4^{(4)}/m'$ (*)		
16	$4/m'$		${}^1D_{4h}''$	
	$4/mm'm'$		${}^2D_{4h}''$	
17	$4'/m'$			
18	$2/m'$			
	$4'/mmm'$			
19	$2'/m$			
20	$6'$	$3^{(3)}$	${}^1D_6''$	
	$62'2'$	$6^{(6)}$		
	$6/m'm'm'$			
21		$6^{(3)}$	${}^2D_6''$	
22	$32'$		D_3''	
	$\bar{6}'m'2$			
	$6'/mmm'$			
23	$6m'm'$		${}^1C_{6v}''$	
	$6/m'mm$		${}^2C_{6v}''$	
24	$\bar{6}'$	$\bar{6}^{(6)}$		
	$6'/m$			
	$6/mm'm'$			
25		$\bar{6}^{(3)}$	${}^2D_{6h}''$	
		$6^{(6)}/m$		

Table 2. (continued)

Composition series number	(1)	(2)	(3)	(4)
26	$6/m'$	$6^{(3)}/m'$	$^1D''_{6h}$	
27	$6'/m'$	$6^{(6)}/m'$	$^4D''_{h}$	
28		$6^{(3)}/m$	$^3D''_{6h}$	
29	$\bar{3}'m'$			
	$6'/m'm'm$			
30	$6'6'2$			
31	$3m'$		C''_{3v}	
	$6'm'm$			
32	$\bar{6}'m2'$			
33	$\bar{6}m'2'$		$^1D''_{3h}$	
			$^2D''_{3h}$	
34	$\bar{3}'$			
35	$\bar{3}'m$			
36	$\bar{3}m'$	$\bar{3}^{(6)}$	$^2D''_{3d}$	
37		$\bar{3}^{(3)}$	$^1D''_{3d}$	
	58	18	23	7

Note to table 2. In column 1 are the double coloured (magnetic) point groups associated with the distinct 1D alternating IRs; in column 2 the polychromatic point groups associated with the pairs of 1D complex IRs; in column 3 the multi coloured point groups associated with the 2D IRs and in column 4, the multi coloured point groups associated with the distinct 3D IRs of the crystallographic point groups are tabulated in the standard notation as given in Shubnikov and Belov (1964), Indenbom *et al'*(1960) and Rama Mohana Rao (1982) respectively.

The double coloured, polychromatic and multi coloured groups enlisted herein are nothing but the minor quasisymmetry (*P*-symmetry) groups with the respective crystallographic point groups G in the considered composition series as generator and with the group of indices (permutations) P chosen to be isomorphic to G by a proper choice of the normal divisors H and Q of G and P respectively.

(*) indicates the operation considered to obtain the particular colour symmetry point group is the quasi semi-direct product.

Case (c): Multi colour point groups

The multi colour point groups are constructed in this section, as a semi-direct product of either a magnetic (double colour) point group or a polychromatic point group (as the case may be) associated with the group G_{i+1} with the colour group generated by the appropriate colour generator in the chosen composition series.

In the case of multi colour groups to be associated with point groups containing one or more 2D IRs, the maximal normal subgroup G_{i+1} of index 2 in the chosen composition series gives us all information, and in the case of multi colour groups with cubic groups G_i containing 3D IRs, either a maximal normal subgroup G_{i+1} of index 3 or of index 2 (depending upon the situation) of G_i suffices our purpose. In all the cases here, the construction is accorded, as in the case of polychromatic point groups of colour value 4 or 6, from the suitably chosen colour group $G_{i+1}^{(p')}$ associated with G_{i+1}. This is illustrated with the help of the series (i).

It is well known that 222 is a normal subgroup of index 3 to 23, with C_{31}^{+} as a generating element of order 3. The magnetic group already associated with 222 denoted by $2'2'2$ is given by E, R_2C_{2x}, C_{2z}, R_2C_{2y} : E, C_{2x} (12), C_{2z}, C_{2y} (12). A double colour group isomorphic with the above† may be viewed as E, C_{2x} (13) (24), C_{2y} (12) (34), C_{2z} (14) (23). C_{31}^+ being an element of order 3, the colour generator associated with it is $R_3C_{31}^+ : C_{31}^+$ (134). The colour group generated with this is E, $R_3C_{31}^+$, $R_3^2C_{31}^-$: E, C_{31}^+ (134), C_{31}^- (143) and this is isomorphic with the polychromatic group $3^{(3)}$ and hence is denoted by $\tilde{3}^{(3)}$. The semi-direct product of $\tilde{3}^{(3)}$ with the group isomorphic to $2'2'2$ suggested above gives rise to the multi colour group associated with 23 which we have denoted by T‴ (Rama Mohana Rao 1982).

In the composition series (i), the polychromatic point group already associated with 23 (generated from 222) is $3^{(3)}/2$. If the generator from 23 to 432 is taken as C_{2a} and the colour group generated with the colour generator R_2C_{2a} is suitably chosen as the double colour group $\tilde{2}^{(2)}$ isomorphic with $2'$ (Rama Mohana Rao 1982), then the semi-direct product of $3^{(3)}/2$ with $\tilde{2}^{(2)}$ gives the multi colour group associated with 432. We identify this group as O″ (Rama Mohana Rao 1982).

Similarly the semi-direct product of T‴ associated with 23 with the group $\tilde{2}^{(2)}$ gives rise to another multi colour group associated with 432 which we have already given the symbol O‴ (Rama Mohana Rao 1982). In a similar way the construction of all the 30 multi colour point groups can be done using the appropriate composition series given in table 1 and the results obtained are enlisted in table 2.

4. The physical significance

The construction of colour symmetry point groups through the method developed here, employing the composition series has physical significance. This can be brought out if the obtained colour symmetry groups generated from various point groups G_{i+1} (or from the colour symmetry groups associated with G_{i+1}) are associated with the appropriate IRs of the group G_i using the AIRs of little groups.

In the case of the non-degenerate IRs, in the little group technique (Bhagavantam and Venkatarayudu 1969), it can be seen that, for the total symmetric IR Γ of the chosen normal subgroup H (here the maximal normal subgroup G_{i+1}), the little group L (G, H, Γ) always coincides with G itself and the kernel (K) coincides with H and thus $L/K \simeq G/H$. Since the IRs of the factor group G/H engender those of the IRs of the same nature of G, the choice of the maximal normal subgroup G_{i+1} of G_i facilitates here the required IR of G_i, which is to be engendered.

For example in the case of (G, H, Γ) = (222, 2, A), the point group G : 222 coincides with the little group L and the group 2 with the kernel K. It can be seen that $222/2 \simeq C_2$ and the alternating IR Γ_2 of the factor group 222/2 engenders the alternating IR B_2 of 222. As $2'2'2$ is generated from the maximal normal subgroup $G_{i+1} = 2$ in the composition series and since the alternating IR of 2 in turn engender the alternating IR of 222, the colour group $2'2'2$ may be associated with the IR B of 222. A similar interpretation holds good for the rest of the magnetic point groups generated in § 3.

To illustrate the case of polychromatic point groups consider $3^{(3)}/2$. It can be seen from § 3 that $3^{(3)}/2$ is associated with 23 and the pair of 1D complex IRs of the factor

† Following the definition of the isomorphism between any two colour symmetry groups as proposed by Bradley and Cracknell (1972) and Krishnamurty *et al* (1978).

group $23/222 \simeq 3$ engenders the complex IRS ${}^1E$ and ${}^2E$ of 23. As $3^{(3)}/2$ is obtained from 222 in the composition series and as the ID complex IRS of the factor group 23/222 (which happens to be the AIR) engender the complex IRS ${}^1E$ and ${}^2E$ of 23, the polychromatic group $3^{(3)}/2$ may be associated with ${}^1E$ of 23 and shown before the point group 23.

So far as the multi colour point groups associated with the degenerate IRS of G_i are concerned, the maximal normal subgroup G_{i+1} of index 2 or 3 (as the case may be) of the group G_i in the series coincides with the little group L and the appropriate IR of G_{i+1} becomes the AIR that induces the respective degenerate IR of G_i. Thus the method of associating a minor quasisymmetry group with the degenerate IR of a point group G_i adopted earlier in respect of the point groups using the little groups and their AIRS (Krishnamurty *et al* 1978, Rama Mohana Rao 1982) can be seen to hold good here in respect of all the multi colour point groups derived through the present method.

It may also be mentioned here that the method of associating the obtained colour symmetry groups through the present method employing the composition series discussed herein satisfies an interesting phenomenon which is involved in the process namely: the number of independent constants required to describe any physical property in respect of any IR of the point group G_i is the same as that occurring for the AIR of the appiopriate little group that engender or induce respectively the corresponding non-degenerate or degenerate IR of the group G_i.

5. Discussion

37 composition series just needed to generate the 106 colour symmetry point groups are constructed here from the 32 crystallographic point groups.

By this method, one need not consider each of the 32 point groups separately for the purpose of constructing each one of the 106 coloured symmetry point groups. They can be obtained in a ladder process.

This method shows, how a subgroup of index 2 of a group G_i induces a magnetic variant of G_i and how one of the colour symmetry groups associated with a maximal normal subgroup G_{i+1} in the series induces another colour symmetry group associated with G_i in that series.

From a considered composition series one can obtain at most one colour symmetry group for each category in respect of any group G_i from its chosen maximal normal subgroup G_{i+1}. The remaining ones, if any, associated with G_i should however be obtained by considering other composition series involving G_i with another suitably chosen maximal normal subgroup G_{i+1}.

If the generating element from the group G_{i+1} to G_i is the same in two distinct composition series, then no new colour symmetry point group associated with G_i can be generated from the subsequent series.

The idea of colour generator, obtained by associating a suitable colour changing operation R_i to the generator g_i (depending upon the order of g_i), from G_{i+1} to G_i in the series can be utilised for the derivation of all the 106 colour symmetry point groups.

The method of generating the different types of colour symmetry groups with a proper choice of the composition series can be extended to the space groups, adopting suitable modifications as and where necessary. Work on these lines is in progress.

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